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WAVERIDERS CONSTRUCTED ON FLOWS FOLLOWING SHOCK WAVES IN THE FORM OF ELLIPTICAL CONES

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In the last few years promising supersonic flight waveriders, which do not have an explicit boundary between carriers and body, are being studied intensively. For the first time the concept of constructing flow over three-dimensional bodies on the basis of two-dimensional shock waves was simultaneously published in [1] for carrying configurations and in [2] for the bodies with minimum resistance. The design of compression surfaces for supersonic flight vehicles using conical shock waves was suggested in [3]. The waverider leading edges are located on a shock wave, while the lower surface coincides with the current lines of supersonic flows passing through this edge. Solving inverse problems of gas dynamics for plane and axisymmetric gas flows, it is possible to construct a wide range of bodies and calculate their aerodynamic characteristics [4-8]. In connection with these works of some interest are analytical [9] and numerical methods calculating flows following curvilinear shock waves without internal shock waves [10, 11].

In this work we present a numerical method for calculating parameters of the flow following arbitrary shock wave and define aerodynamic characteristics of the waveriders, the compression surfaces of which are constructed on the flow following the shock waves in the form of elliptical cones. The upper surfaces of waveriders are the planes which intersect at the angle of divergence and are parallel to the free stream velocity vector. Calculating aerodynamic characteristics, we neglected the friction forces and set the bottom pressure to be equal to the free stream pressure, as well as to zero.

1. Calculational Technique for Flow Parameters. Let the surface shape of an arbitrary adjoint shock wave be known. The equation of this wave in the rectangular coordinate system XYZ, where the X-axis is directed parallel to the free stream velocity vector and the Y-axis is directed upward, has the form $F(x, y, z) = 0$ (Fig. 1a, b). We write the system of equations of gas dynamics for a steady-state flow of an ideal liquid: a continuity equation, three equations of motion, and equation of constant entropy along flow lines. This system is completed with ten equations, which help to calculate the parameters of flow along two known directions on the given surface. For this surface we choose a shock wave, and then we obtain a system of fifteen algebraic equations with respect to the first derivatives of the flow parameters on the shock wave (the flow parameters themselves can be calculated from the angle of wave inclination to the X-axis).

The design network (coordinates of flow lines following the shock wave) is formed as follows. The shock wave shape is set and its section is selected beginning from the leading edge of the waverider being formed (the front point of the intersection line of its upper surface and the shock wave) and the length along the free stream velocity vector equal to unity (this length can be chosen arbitrarily). The flow line starting points ($i = 1$) are located on the lower part of the shock wave, which is cut off by the waverider's upper surface from its remaining part.

Let us restrict ourselves by a part of this wavelength along the X-axis equal to unity. We separate it by N planes which are perpendicular to the X-axis and divide this part into small equal intervals of length Δx_0. We draw through the points of their intersection with the shock wave in the plane of symmetry N surfaces parallel to the upper surface. We denote the lines of their intersection with the wave by the index k ($k = 1$ corresponds to the leading edge) and the point of intersection of these lines with the planes perpendicular to the X-axis, by j. Thus, a network of calculated points has been constructed on a given shock wave (Fig. 1a, where 1 is the shock wave, 2 is the leading edge ($k = 1$), 3 are the lines $k = \text{const}$, 4 are the design points). Let us enumerate the flow parameters at the design points by i, j, and k, where $i = 1$ corresponds to the design points on the shock wave surface.

Let us write the first and the last equations from the system of fifteen algebraic equations with respect to the derivatives of the flow parameters at the design points:

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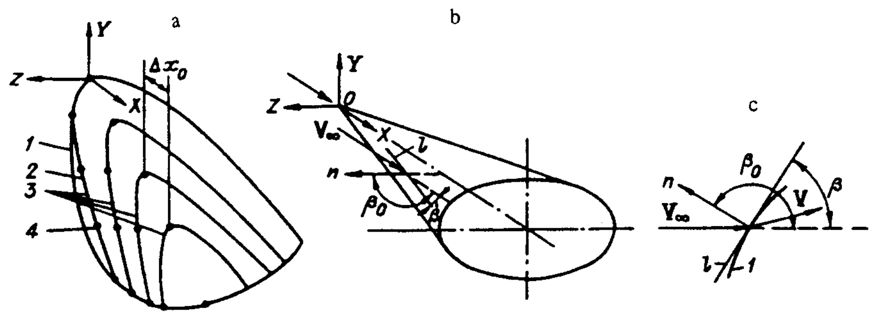

Fig. 1

$$V_{xijk}\left(\frac{\partial\rho}{\partial x}\right)_{ijk} + \rho_{ijk}\left(\frac{\partial V_x}{\partial x}\right)_{ijk} + V_{yijk}\left(\frac{\partial\rho}{\partial y}\right)_{ijk} + \rho_{ijk}\left(\frac{\partial V_y}{\partial y}\right)_{ijk} + V_{zijk}\left(\frac{\partial\rho}{\partial z}\right)_{ijk} + \rho_{ijk}\left(\frac{\partial V_z}{\partial z}\right)_{ijk} = 0,$$

$$\ldots \qquad (1.1)$$

$$\cos(L_2, x)_{ijk}\left(\frac{\partial\rho}{\partial x}\right)_{ijk} + \cos(L_2, y)_{ijk}\left(\frac{\partial\rho}{\partial y}\right)_{ijk} + \cos(L_2, z)_{ijk}\left(\frac{\partial\rho}{\partial z}\right)_{ijk} = \left(\frac{\partial\rho}{\partial L_2}\right)_{ijk}.$$

The derivatives of the flow parameters g_n in the direction of L_1 (connecting points k along the tangent line to the shock wave) and L_2 (connecting the points g), and the direction cosines of these directions are calculated from the equations

$$\left(\frac{\partial g_n}{\partial L_1}\right)_{ijk} = (g_{nij+1k} - g_{nijk})/R_{ijk},$$

$$R_{ijk} = [\Delta x_0^2 + (y_{ij+1k} - y_{ijk})^2 + (z_{ij+1k} - z_{ijk})^2]^{0.5},$$

$$\cos(L_1, x)_{ijk} = \Delta x_0/R_{ijk},$$

$$\cos(L_1, y)_{ijk} = (y_{ij+1k} - y_{ijk})/R_{ijk},$$

$$\cos(L_1, z)_{ijk} = (z_{ij+1k} - z_{ijk})/R_{ijk},$$

$$\left(\frac{\partial g_n}{\partial L_2}\right)_{ijk} = (g_{nijk+1} - g_{nijk})/R_{0ijk},$$

$$R_{0ijk} = [\Delta x_0^2 + (y_{ijk+1} - y_{ijk})^2 + (z_{ijk+1} - z_{ijk})^2]^{0.5}, \quad \cos(L_2, x)_{ijk} = \Delta x_0/R_{0ijk},$$

$$\cos(L_2, y)_{ijk} = (y_{ijk+1} - y_{ijk})/R_{0ijk}, \qquad \cos(L_2, z)_{ijk} = (z_{ijk+1} - z_{ijk})/R_{0ijk}.$$

Solving the system of equations (1.1) for i = const for each value of j and k ($1 \le j \le N - i + 1$, $1 \le k \le N - i + 1$), we get the first derivatives of the flow parameters. To calculate the second derivatives we differentiate each equation of (1.1) with respect to X, Y, and Z. Then we get a system of 45 equations with respect to 30 second derivatives of the flow parameters and first derivatives of the direction cosines of the known directions L_1 and L_2. Since the number of the first derivatives of the direction cosines is 18, the system of 45 equations is added by the first three derivatives of X, Y, and Z of any direction cosine, the derivatives being calculated in the nodes of the design network. It was verified that the values of the remaining derivatives of the direction cosines, calculated at the design points as the unknowns in the system of 48 algebraic equations, differed from that calculated by a direct method by less than 0.2% for N = 120.

To calculate the derivatives of the direction cosines along any direction; the corresponding points for i > 1 were connected by parabolas by three points with a step equal to the distance between two adjacent points.

The derivatives of the flow parameters of any order can be calculated in similar manner. Then the required flow parameters are calculated by Taylor's formula on the second (i + 1)st step by their known values and the values of the derivatives on the previous i-th step. The current lines between two adjacent design points (i-th and (i+1)st) are set to be rectilinear and the velocities between these points to be constant and equal in magnitude to the velocities calculated at the previous i-th step.

For the solution stability the increments along the axis X(Δx_{ijk}) were chosen so that the Mach cone of the velocity V_{ijk} at the point (i + 1, j, k) touch a rectilinear segment that connects the points (i, j + 1, k) and (i, j, k + 1). For the case when

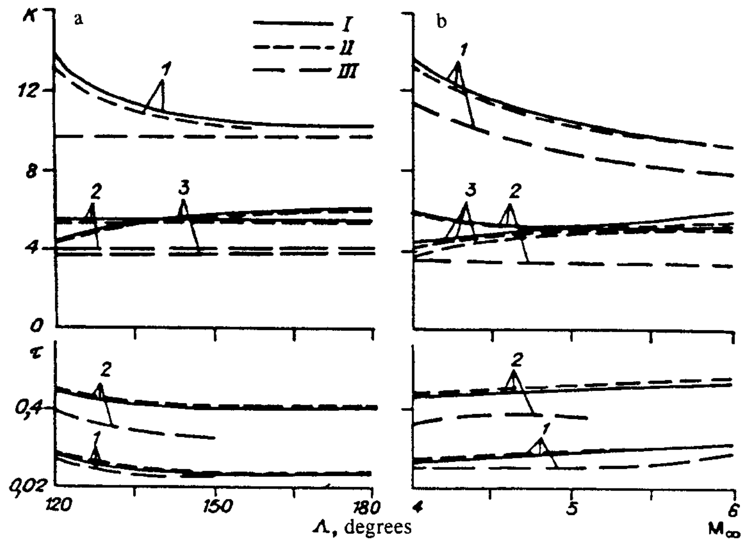

Fig. 2

the point was outside the given segment, it was provided that the Mach cone touched the nearest of the points (i, j + 1, k) or (i, j, k+1).

In order to solve the posed problems it is necessary to know the flow parameters on the shock wave according to their values in a free flow. At first the inclination angle of the shock wave to the X-axis was calculated at an arbitrary point (Fig. 1b, c, where l is the tangent line to the wave shock in the plane V_∞, n, V is the velocity following the shock wave, and β is the angle between the shock and the vector of the incoming flow in the plane of the velocity vector of the incoming flow and the normal to the wave shock). The direction cosines of the external normal n to the shock surface and of the velocity vector of the free flow have the form

$$l_n = (\partial F/\partial x)/E,\ m_n = (\partial F/\partial y)/E,\ n_n = (\partial F/\partial z)/E,$$
$$E = \left[\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2\right]^{0.5},$$
$$l_V = 1,\ m_V = 0,\ n_V = 0.$$

Then the angle of inclination of the shock (the angle of inclination of the tangent line to the shock, which lies in the plane of the free flow velocity vector and normal vector, with respect to the free flow velocity vector)

$$\beta = \beta_0 - \frac{\pi}{2}$$
$$(\beta_0 = \arccos(l_n l_V + m_n m_V + n_n n_V)).$$

In order to determine the velocity vector components on the shock with the direction cosines l_0, m_0, n_0 ($l_0 = \cos\delta$, δ is the angle of the flow deviation upon the passage of the shock wave calculated from the angle β) a unit vector is introduced, which is perpendicular to the plane passing through V_∞ and n. Its direction cosines l_1, m_1, n_1 are determined from the system of equations

$$l_1 \cdot l_n + m_1 \cdot m_n + n_1 \cdot n_n = 0,$$
$$l_1 \cdot 1 + m_1 \cdot 0 + n_1 \cdot 0 = 0,\ l_1^2 + m_1^2 + n_1^2 = 1.$$

Then

$$l_0 \cdot 0 + m_0 \cdot m_1 + n_0 \cdot n_1 = 0,\ m_0^2 + n_0^2 = 1 - l_0^2.$$

The components of the velocity vector on the shock wave are $V_x = Vl_0$, $V_y = Vm_0$, $V_z = Vn_0$.

2. Aerodynamic Characteristics of Waveriders. Aerodynamic characteristics of waveriders I with the compression surfaces constructed on the flows following the shock waves in the form of elliptical cones were calculated. The following equations have been obtained:

$$z^2/A_1^2 + y^2/B_1^2 = x^2.$$

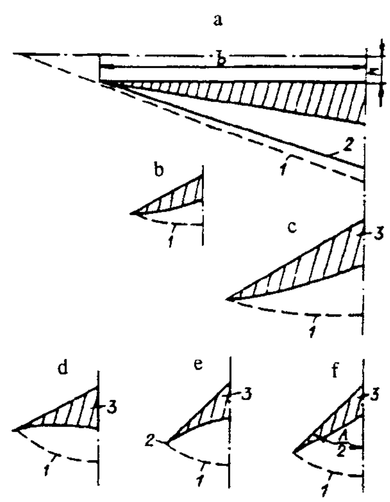

Fig. 3

The upper surfaces consisted of the planes intersecting at an angle directed parallel to the free flow velocity vector. A maximum relative difference of the parameters of the flow following these shock waves, calculated with an accuracy to the first-order derivatives for N = 70 was ~9% of the corresponding values for N = 40, and for N = 100 was ~3% of the value for N = 70. For N = 100 the flow parameters calculated with an accuracy to the second-order derivatives differed by less than 2.5% of the corresponding values calculated with an accuracy to the first-order derivatives. Therefore, all further calculations were carried out for N = 100 with an accuracy to the first-order derivatives.

Figure 2 presents the results of calculations of aerodynamic quality K and volume coefficient $\tau = W/S^{3/2}$ (W is the volume and S is the area on the plane) of the waveriders constructed on the flow in the form of circular cones depending on the angle of divergence Λ(a) and the Mach number (b) for $c_{ya} = 0.06$, θ_s 9.5° and $c_{ya} = 0.2$, and $\theta_s = 18°$ (lines 1 and 2), here the bottom pressure was set equal to the free flow pressure (the case (a) corresponds to $M_\infty = 4$, h/b = 0.25, $\Lambda = 120°$). Number 3 denotes similar calculations with the bottom pressure equal to zero for $c_{ya} = 0.06$, $\theta_s = 9.5°$, I is the waverider with axisymmetric conical shock, II is the waverider constructed by the method [3], III is the waverider with a flat shock. As it follows from the figure, the characteristics of waverider I constructed by the given method are in good agreement with aerodynamic characteristics calculated by an exact method [3].

Figure 3a-c shows the cross sections of waveriders I constructed on the flow following the shock wave in the form of elliptical cone (1 is the shock wave, 2 is the leading edge, 3 is the bottom cut, $M_\infty = 6$, $A_1 = 0.534$, $A_1/B_1 = 1.7$, h/b = 0.6, $\Lambda = 120°$; a is the cross section by the plane of symmetry, b is the central cross section, and c is the bottom cut). The bottom cuts of waveriders II, constructed on the flow following the shock waves in the form of circular cones, are given in Fig. 3e-g [e) $M_\infty = 6$, $\theta_s = 9.5°$, $\Lambda = 120°$, h/b = 0.25; f) $M_\infty = 4$, $\theta_s = 10°$, $\Lambda = 90°$, h/b = 0,25; g) $M_\infty = 4$, $\theta_s = 10°$, $\Lambda = 90°$, h/b = 0.3 (θ_s is a semi-angle of the cone opening, which forms a shock wave)]. It is seen that the shapes of the waverider bottom sections constructed on the flows, following the shock waves in the form of circular cones, depend essentially on the ratio h/b.

The dependences of aerodynamic characteristics of waveriders I and III, similar by the angle of divergence Λ, ratio h/b ($\Lambda = 120°$, h/b = 0,25), volume coefficient or coefficient of lift, on M_∞ are given in Fig. 4 (the bottom pressure was considered to be equal to the pressure of the free flow). The calculations are carried out for three versions of the shock wave 1 – $A_1 = 0.314$, $B_1 = 0.449$; 2 – 0.314; 0.314; 3 – 0.44; 0.314. For equal values of the coefficient of lift or volume coefficient the aerodynamic quality of waveriders I, constructed on the flows following the shock waves in the form of the elliptical cones, exceeds the aerodynamic quality of waveriders III, whose compression surfaces form a plane shock wave.

Figure 5 presents the same aerodynamic characteristics with respect to the ratio between the semiaxes of elliptical shock A_1/B_1 ($M_\infty = 5$, $\Lambda = 120°$, $B_1 = 0.314$). Lines 1-3 correspond to h/b = 0.25; 0.5; 1.0. The calculations carried out for waveriders I and waveriders III, which are equivalent in the coefficient of lift waveriders III. It is seen that aerodynamic quality of waveriders increases with A_1/B_1 and the coefficients of lift and volume decreases.

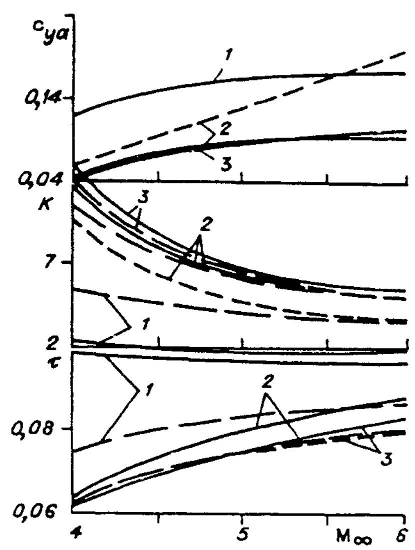

Fig. 4

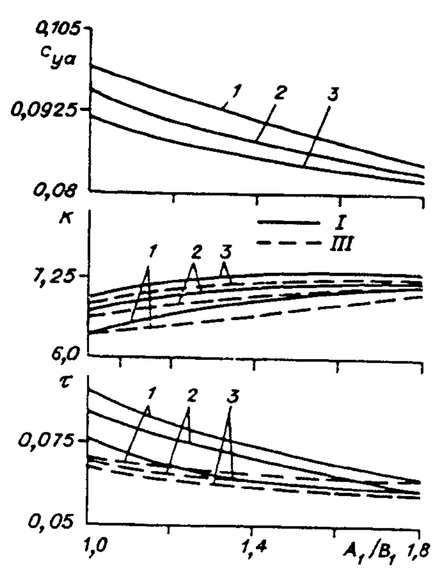

Fig. 5

We have also defined the aerodynamic characteristics of waveriders (whose compression surfaces are constructed on the flows following the shock waves in the form of the surfaces with elliptical cross sections) with concave (waveriders IV) and convex (waveriders V) generatrices, which change according to the power law. The calculations have shown that for a given value of the coefficient of lift as well as the volume coefficient, waverider I with a straight generatrix has a great excess of aerodynamic quality with respect to waveriders III in the class of forms of waveriders I, IV, and V.

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